Total No. of Questions : 9]

SEAT No. :

P9084

[Total No. of Pages : 5

[6179]-209

S.E. (Civil)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No.1 is compulsory. Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Figures to the right indicate full marks.*
3) *Non-programmable electronic pocket calculator is allowed.*
4) *Assume suitable data, if necessary.*
5) *Neat diagrams must be drawn wherever necessary.*

Q1) Attempt the following.

a) If $\sum xy = 2638, \bar{x} = 14, \bar{y} = 17, n = 10$, then cov (x, y) is_____ **[2]**

i) 24.2 ii) 25.8

iii) 23.9 iv) 20.5

b) If $\overline{F} = r^2\bar{r}$ then $\overline{F}$ is_____ **[2]**

i) Constant ii) Conservative

iii) Solenoidal iv) None of these

c) For $\overline{F} = x^2\hat{i} + xy\ \hat{j}$, the value of $\int_c \overline{F} - d\bar{r}$ for curve $y^2 = x$ joining points (0, 0) and (1, 1) is_____ **[2]**

i) 1 ii) $\frac{1}{3}$

iii) $\frac{3}{2}$ iv) $\frac{2}{3}$

d) General solution of PDE $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ is___ **[2]**

i) $u(x, t) = (C_4 \cos mx + C_5 \sin mx)\, e^{-4m^2t}$

ii) $u(x, t) = (C_1 \cos mx + C_2 \sin mx)\, (C_3 \cos 2mt + C_4 \sin 2mt)$

iii) $u(x, y) = (C_1 e^{mx} + C_2 \bar{e}^{mx})\, (C_3 \cos my + C_4 \sin my)$

iv) $u(x, y) = (C_1 \cos mx + C_2 \sin mx)\, (C_3 e^{my} + C_4 \bar{e}^{my})$

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e) Coefficient of kurtosis β_2 is given by **[1]**

i) $\frac{\mu_4}{\mu_3}$ ii) $\frac{\mu_4}{\mu_2^2}$

iii) $\frac{\mu_3}{\mu_2^2}$ iv) $\frac{\mu_4}{\mu_2^3}$

f) The dot product of two vectors $\bar{a}$ & $\bar{b}$ is defined as $\bar{a}.\bar{b}$ =_______. **[1]**

i) ab cosθ ii) ab sinθ

iii) ab sinθ $\hat{n}$ iv) ab

Q2) a) The first four moments of a distribution about value 5 are 2, 20, 40 and 50. From given information find first four central moments. Also find coefficient of skewness and kurtosis. **[5]**

b) Find the coefficient of correlation for the following data. **[5]**

x	y
10	18
14	12
18	24
22	6
26	30
30	36

c) Between 2. p.m and 3.p.m the average number of phone calls per minute coming into company are 2. Find probability that during one particular minute there will be 2 or less calls. **[5]**

OR

Q3) a) Given the following information. **[5]**

	Variable x	Variable y
Arithmetic mean	8.2	12.4
Standard deviation	6.2	20

Coefficient of correlation between x & y is 0.9. Find the libean regression estimate of x, given y = 10.

b) On an average a box containing 10 articles is likely to have 2 detectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less detectives? **[5]**

c) In a normal distribution 10% of items are under 40 and 5% are over 80. Find mean and standard deviation of distribution. [Given : A(1.29) = 0.4, A(1.65) = 0.45] **[5]**

Q4) a) Find the angle between tangents to the curve $\vec{r} = (t^3+2)\hat{i} + (4t-2)\hat{j} + (2t^2-6t)\hat{k}$ at t = 0 and t = 2. **[5]**

b) Find the directional derivative of $\phi = x^2y + xyz + z^3$ at (1, 2, –1) along normal to the surface $x^2 + y^2 + z^2 = 9$ at the point (1, 2, 0). **[5]**

c) Show that $\vec{F} = (ye^{xy}\cos z)\hat{i} + (xe^{xy}\cos z)\hat{j} - e^{xy}\sin z\,\hat{k}$ is irrotational. Find corresponding scalar & such that $\vec{F} = \nabla\phi$ **[5]**

OR

Q5) a) If the directional derivative of $\phi = a(x + y) + b(y + z) + c(x + z)$ has maximum value 12 in the direction parallel to y axis. Find a, b and c. **[5]**

b) Attempt any one. **[5]**

i) $\nabla \times \left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a}.\vec{r})}{r^5}\vec{r}$

ii) $\nabla^4 e^r = e^r + \frac{4}{r}e^r$

c) Show that the vector field $f(r)\vec{r}$ is always irrotational and determine $f(r)$ such that the field is solenoidal. **[5]**

Q6) a) Let $\overline{F} = (xy + y^2)\hat{i} + x^2\hat{j}$. Is the work done along $y = x$ and $y = x^2$ from the common starting point (0, 0) to the common and point (1, 1), the same or different? **[5]**

b) Evaluate $\iint_s \overline{F}.\hat{n}\,dS$ where $\overline{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$ and S in the surface of the sphere $x^2 + y^2 + z^2 = r^2$. **[5]**

c) Apply stokes theorem to evaluate

$$\int_C \left[(x+y)dx + (2x-z)dy + (y+z)dz\right]$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). **[5]**

OR

Q7) a) Evaluate $\oint_C \left[(3x-y)dx+(2x+y)dy\right]$ applying Green's lemma where C is the curve $x^2 + y^2 = a^2$. Is the work done the same along the curves C_1 and C_2 where C_1 is the arc of C from (0, –1) to (0, 1) clockwise and C_2 is the arc of C from (0, –1) to (0, 1) anti clockwise. **[5]**

b) Let S be the surface of the sphere $(z + 3)^2 + x^2 + y^2 = 4^2$ cut off by the plane z = –2. Evaluate $\iint_s \nabla \times \overline{F}.d\overline{S}$ where

$\overline{F}=(x+y)\hat{i}+(y+z)\hat{j}+(z+x)\hat{k}$ **[5]**

c) Find the surface of equi pressure in case of steady motion of a liquid which has velocity potential $\phi = \log(xyz)$ and is under the action of force $\overline{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$. Use the equation

$\frac{\partial \overline{q}}{\partial t}+\frac{1}{2}\nabla q^2=-\nabla v-\frac{1}{p}\nabla p$ assigning appropriate meanings to the variables. **[5]**

Q8) a) Solve the equation, $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$, where $y(x, t)$ satisfies the following conditions, **[8]**

i) $y(0, t) = 0 \;\forall t$

ii) $y(L, t) = 0 \;\forall t$

iii) $\left.\frac{\partial y}{\partial t}\right|_{t=0}=0\;\forall x$

iv) $y(x,0)=a\sin\left(\frac{\pi x}{L}\right)$

b) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, with conditions, **[7]**

i) $u = 0$ as $y\to\infty\;\forall x$

ii) $u = 0$ at $x=0\;\forall y$

iii) $u = 0$ at $x=\pi\;\forall y$

iv) $u = u_0$ at $y=0, 0<x<\pi$

OR

Q9) a) A tightly stretched string with fixed ends $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l - x)$ for each $0 < x < l$. Find the displacement $y(x, t)$. **[8]**

b) Solve, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if, **[7]**

i) u is finite for all l

ii) $u(0, t) = 0$

iii) $u(l, t) = 0$

iv) $u(x, 0) = \frac{3x}{l}$ $0 \le x \le l$

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